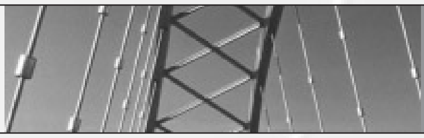


# 12

# Review of Centroids and Moments of Inertia

## Differential Equations of the Deflection Curve

*The problems for Section 12.2 are to be solved by integration.*

**Problem 12.2-1** Determine the distances $\bar{x}$ and $\bar{y}$ to the centroid $C$ of a right triangle having base $b$ and altitude $h$ (see Case 6, Appendix D).

**Solution 12.2-1 Centroid of a right triangle**

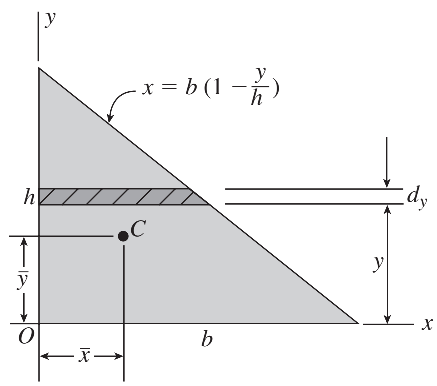
$dA = x\,dy = b(1 - y/h)\,dy$

$$A = \int dA = \int_0^h b(1 - y/h)\,dy = \frac{bh}{2}$$

$$Q_x = \int y\,dA = \int_0^h yb(1 - y/h)\,dy = \frac{bh^2}{6}$$

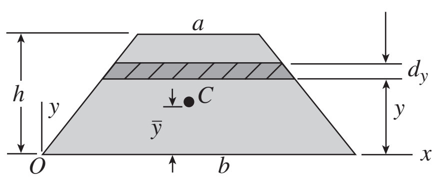
$$\bar{y} = \frac{Q_x}{A} = \frac{h}{3} \quad \longleftarrow$$

Similarly, $\bar{x} = \dfrac{b}{3}$ $\longleftarrow$




**Problem 12.2-2** Determine the distance $\bar{y}$ to the centroid $C$ of a trapezoid having bases $a$ and $b$ and altitude $h$ (see Case 8, Appendix D).

**Solution 12.2-2 Centroid of a trapezoid**




Width of element $= b + (a - b)y/h$

$dA = [b + (a - b)y/h]\,dy$

$$A = \int dA = \int_0^h [b + (a - b)y/h]\,dy = \frac{h(a + b)}{2}$$

$$Q_x = \int y\,dA = \int_0^h y[b + (a - b)\,y/h]dy$$

$$= \frac{h^2}{6}(2a + b)$$

$$\bar{y} = \frac{Qx}{A} = \frac{h(2a + b)}{3(a + b)} \quad \longleftarrow$$

**Problem 12.2-3** Determine the distance $\bar{y}$ to the centroid $C$ of a semicircle of radius $r$ (see Case 10, Appendix D).

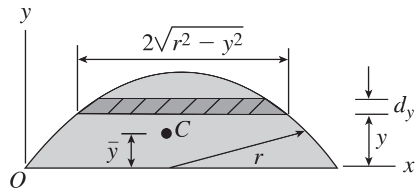
### Solution 12.2-3 Centroid of a semicircle

$$dA = 2\sqrt{r^2 - y^2}\,dy$$

$$A = \int dA = \int_0^r 2\sqrt{r^2 - y^2}\,dy = \frac{\pi r^2}{2}$$

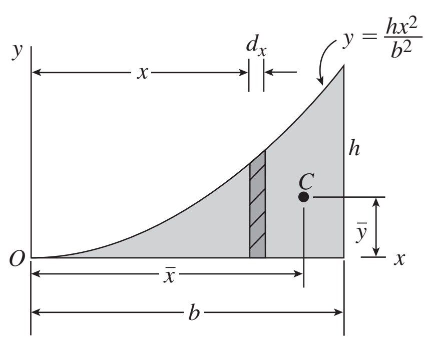
$$Q_x = \int y\,dA = \int_0^r 2y\sqrt{r^2 - y^2}\,dy = \frac{2r^3}{3}$$

$$\bar{y} = \frac{Q_x}{A} = \frac{4r}{3\pi} \quad \longleftarrow$$




**Problem 12.2-4** Determine the distances $\bar{x}$ and $\bar{y}$ to the centroid $C$ of a parabolic spandrel of base $b$ and height $h$ (see Case 18, Appendix D).

### Solution 12.2-4 Centroid of a parabolic spandrel




$$dA = y\,dx = \frac{hx^2\,dx}{b^2}$$

$$A = \int dA = \int_0^b \frac{hx^2}{b^2}\,dx = \frac{bh}{3}$$

$$Q_y = \int x\,dA = \int_0^b \frac{hx^3}{b^2}\,dx = \frac{b^2h}{4}$$

$$\bar{x} = \frac{Q_y}{A} = \frac{3b}{4} \quad \longleftarrow$$

$$Q_x = \int y/2\,dA = \int_0^b \frac{1}{2}\left(\frac{hx^2}{b^2}\right)\left(\frac{hx^2}{b^2}\right)dx = \frac{bh^2}{10}$$

$$\bar{y} = \frac{Q_x}{A} = \frac{3h}{10} \quad \longleftarrow$$

**Problem 12.2-5** Determine the distances $\bar{x}$ and $\bar{y}$ to the centroid $C$ of a semisegment of $n$th degree having base $b$ and height $h$ (see Case 19, Appendix D).

### Solution 12.2-5 Centroid of a semisegment of *n*th degree

$$dA = y\,dx = h\left(1 - \frac{x^n}{b^n}\right)dx$$

$$A = \int dA = \int_0^b h\left(1 - \frac{x^n}{b^n}\right)dx = bh\left(\frac{n}{n+1}\right)$$

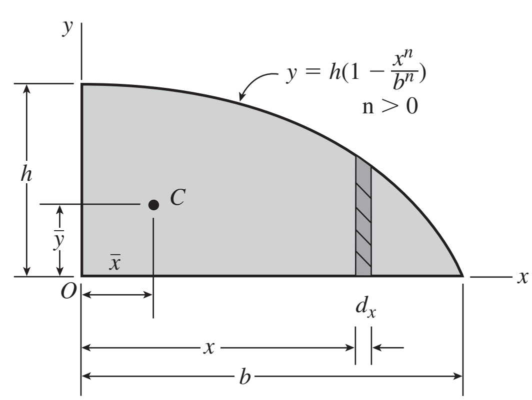
$$Q_y = \int x\,dA = \int_0^b xh\left(1 - \frac{x^n}{b^n}\right)dx = \frac{hb^2}{2}\left(\frac{n}{n+2}\right)$$

$$\bar{x} = \frac{Q_y}{A} = \frac{b(n+1)}{2(n+2)} \quad \longleftarrow$$

$$Q_x = \int \frac{y}{2}\,dA = \int_0^b \frac{1}{2}h\left(1 - \frac{x^n}{b^n}\right)(h)\left(1 - \frac{x^n}{b^n}\right)dx$$

$$= bh^2\left[\frac{n^2}{(n+1)(2n+1)}\right]$$

$$\bar{y} = \frac{Q_x}{A} = \frac{hn}{2n+1} \quad \longleftarrow$$

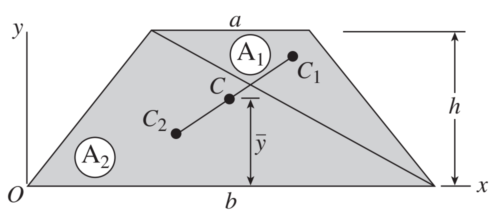


## Centroids of Composite Areas

*The problems for Section 12.3 are to be solved by using the formulas for composite areas.*

**Problem 12.3-1** Determine the distance $\bar{y}$ to the centroid $C$ of a trapezoid having bases $a$ and $b$ and altitude $h$ (see Case 8, Appendix D) by dividing the trapezoid into two triangles.

### Solution 12.3-1 Centroid of a trapezoid



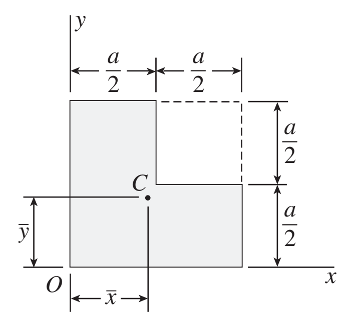

$$A_1 = \frac{ah}{2} \qquad \bar{y}_1 = \frac{2h}{3} \qquad A_2 = \frac{bh}{2} \qquad \bar{y}_2 = \frac{h}{3}$$

$$A = \sum A_i = \frac{ah}{2} + \frac{bh}{2} = \frac{h}{2}(a + b)$$

$$Q_x = \sum \bar{y}_i A_i = \frac{2h}{3}\left(\frac{ah}{2}\right) + \frac{h}{3}\left(\frac{bh}{2}\right) = \frac{h^2}{6}(2a + b)$$

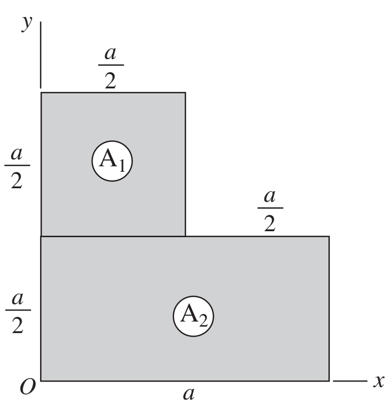
$$\bar{y} = \frac{Q_x}{A} = \frac{h(2a + b)}{3(a + b)} \quad \longleftarrow$$

**Problem 12.3-2** One quarter of a square of side $a$ is removed (see figure). What are the coordinates $\bar{x}$ and $\bar{y}$ of the centroid $C$ of the remaining area?




**PROBS. 12.3-2 and 12.5-2**

### Solution 12.3-2 Centroid of a composite area




$$A_1 = \frac{a^2}{4} \qquad \bar{y}_1 = \frac{3a}{4}$$

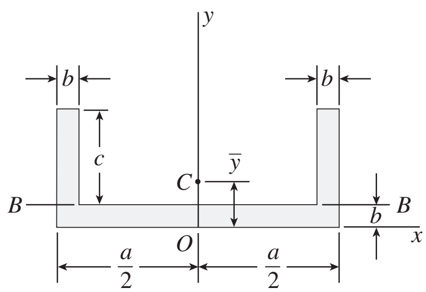
$$A_2 = \frac{a^2}{2} \qquad \bar{y}_2 = \frac{a}{4}$$

$$A = \sum A_i = \frac{3a^2}{4}$$

$$Q_x = \sum \bar{y}_i A_i = \frac{3a}{4}\left(\frac{a^2}{4}\right) + \frac{a}{4}\left(\frac{a^2}{2}\right) = \frac{5a^3}{16}$$

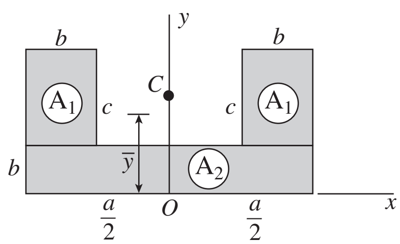
$$\bar{x} = \bar{y} = \frac{Qx}{A} = \frac{5a}{12} \quad \longleftarrow$$

**Problem 12.3-3** Calculate the distance $\bar{y}$ to the centroid $C$ of the channel section shown in the figure if $a = 6$ in., $b = 1$ in., and $c = 2$ in.




**PROBS. 12.3-3, 12.3-4, and 12.5-3**

## Solution 12.3-3 Centroid of a channel section




$a = 6$ in. $\quad b = 1$ in. $\quad c = 2$ in.

$$A_1 = bc = 2 \text{ in.}^2 \qquad \bar{y}_1 = b + c/2 = 2 \text{ in.}$$

$$A_2 = ab = 6 \text{ in.}^2 \qquad \bar{y}_2 = \frac{b}{2} = 0.5 \text{ in.}$$

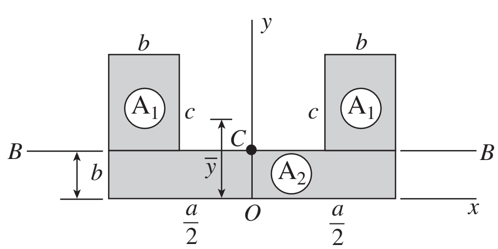
$$A = \sum A_i = 2A_1 + A_2 = 10 \text{ in.}^2$$

$$Q_x = \sum \bar{y}_i A_i = 2\bar{y}_1 A_1 + \bar{y}_2 A_2 = 11.0 \text{ in.}^3$$

$$\bar{y} = \frac{Q_x}{A} = 1.10 \text{ in.} \quad \longleftarrow$$

**Problem 12.3-4** What must be the relationship between the dimensions $a$, $b$, and $c$ of the channel section shown in the figure in order that the centroid $C$ will lie on line $BB$?

## Solution 12.3-4 Dimensions of channel section




$$A_1 = bc \qquad \bar{y}_1 = b + c/2$$

$$A_2 = ab \qquad \bar{y}_2 = b/2$$

$$A = \sum A_i = 2A_1 + A_2 = b(2c + a)$$

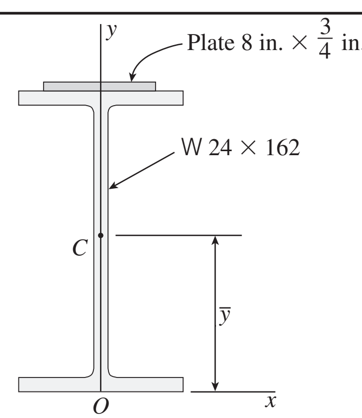
$$Q_x = \sum \bar{y}_i A_i = 2\bar{y}_1 A_1 + \bar{y}_2 A_2 = b/2(4bc + 2c^2 + ab)$$

$$\bar{y} = \frac{Q_x}{A} = \frac{4bc + 2c^2 + ab}{2(2c + a)}$$

Set $\bar{y} = b$ and solve: $2c^2 = ab \quad \longleftarrow$

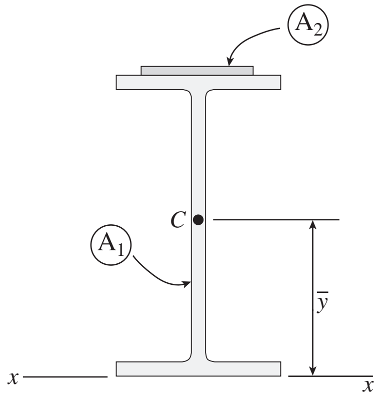
**Problem 12.3-5** The cross section of a beam constructed of a W 24 × 162 wide-flange section with an 8 in. × 3/4 in. cover plate welded to the top flange is shown in the figure.

Determine the distance $\bar{y}$ from the base of the beam to the centroid $C$ of the cross-sectional area.




**PROBS. 12.3-5 and 12.5-5**

**Solution 12.3-5** **Centroid of beam cross section**




W 24 × 162 $A_1 = 47.7 \text{ in.}^2$ $d = 25.00$ in.

$\bar{y}_1 = d/2 = 12.5$ in.

PLATE: 8.0 × 0.75 in. $A_2 = (8.0)(0.75) = 6.0 \text{ in.}^2$

$\bar{y}_2 = 25.00 + 0.75/2 = 25.375$ in.

$A = \sum A_i = A_1 + A_2 = 53.70 \text{ in.}^2$

$Q_x = \sum \bar{y}_i A_i = \bar{y}_1 A_1 + \bar{y}_2 A_2 = 748.5 \text{ in.}^3$

$\bar{y} = \dfrac{Q_x}{A} = 13.94$ in. ←

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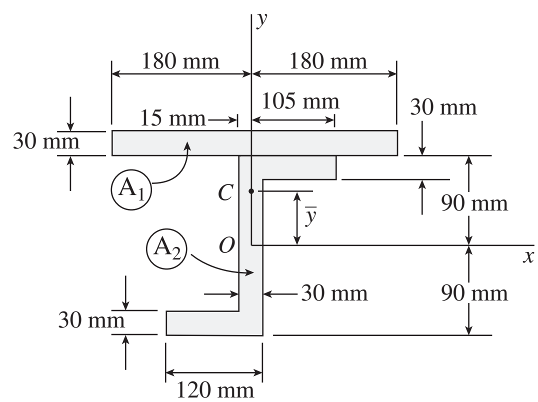
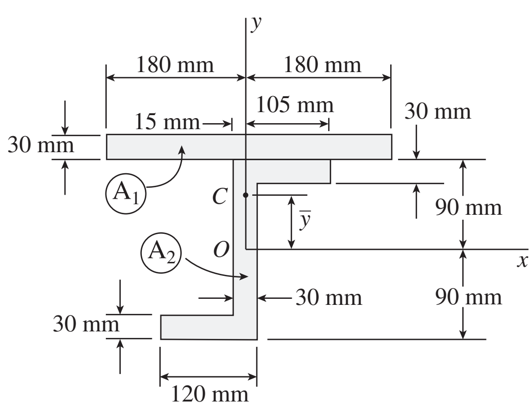
**Problem 12.3-6** Determine the distance $\bar{y}$ to the centroid $C$ of the composite area shown in the figure.




**PROBS. 12.3-6, 12.5-6 and 12.7-6**

**Solution 12.3-6** **Centroid of composite area**




$A_1 = (360)(30) = 10{,}800 \text{ mm}^2$

$\bar{y}_1 = 105$ mm

$A_2 = 2(120)(30) + (120)(30) = 10{,}800 \text{ mm}^2$

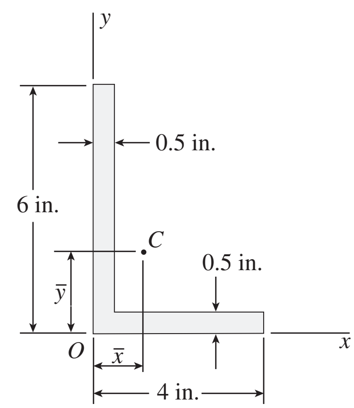
$\bar{y}_2 = 0$

$A = \sum A_i = A_1 + A_2 = 21{,}600 \text{ mm}^2$

$Q_x = \sum \bar{y}_i A_i = \bar{y}_1 A_1 + \bar{y}_2 A_2 = 1.134 \times 10^6 \text{ mm}^3$

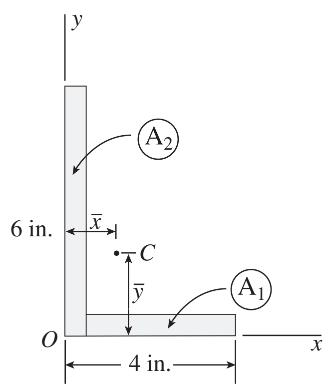
$\bar{y} = \dfrac{Q_x}{A} = 52.5$ mm ←

**Problem 12.3-7** Determine the coordinates $\bar{x}$ and $\bar{y}$ of the centroid $C$ of the L-shaped area shown in the figure.




**PROBS. 12.3-7, 12.4-7, 12.5-7 and 12.7-7**

**Solution 12.3-7** **Centroid of L-shaped area**




$A_1 = (3.5)(0.5) = 1.75 \text{ in.}^2$

$\bar{y}_1 = 0.25 \text{ in.} \quad \bar{x}_1 = 2.25 \text{ in.}$

$A_2 = (6)(0.5) = 3.0 \text{ in.}^2$

$\bar{y}_2 = 3.0 \text{ in.} \quad \bar{x}_2 = 0.25 \text{ in.}$

$$A = \sum A_i = A_1 + A_2 = 4.75 \text{ in.}^2$$

$$Q_y = \sum \bar{x}_i A_i = \bar{x}_1 A_1 + \bar{x}_2 A_2 = 4.688 \text{ in.}^3$$

$$\bar{x} = \frac{Q_y}{A} = 0.99 \text{ in.} \quad \longleftarrow$$

$$Q_x = \sum \bar{y}_i A_i = \bar{y}_1 A_1 + \bar{y}_2 A_2 = 9.438 \text{ in.}^3$$

$$\bar{y} = \frac{Q_x}{A} = 1.99 \text{ in.} \quad \longleftarrow$$

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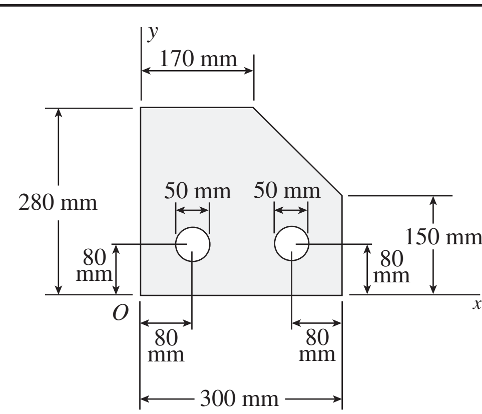
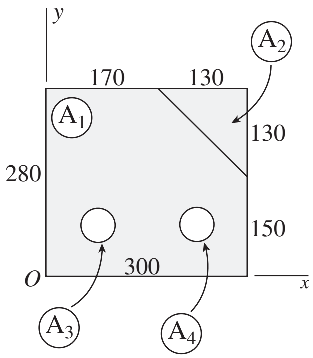
**Problem 12.3-8** Determine the coordinates $\bar{x}$ and $\bar{y}$ of the centroid $C$ of the area shown in the figure.



**Solution 12.3-8** **Centroid of composite area**




$A_1$ = large rectangle
$A_2$ = triangular cutout
$A_3 = A_4$ = circular holes
All dimensions are in millimeters.
Diameter of holes = 50 mm
Centers of holes are 80 mm from edges.

$A_1 = (280)(300) = 84{,}000 \text{ mm}^2$

$\bar{x}_1 = 150 \text{ mm} \qquad \bar{y}_1 = 140 \text{ mm}$

$A_2 = 1/2(130)^2 = 8450 \text{ mm}^2$

$\bar{x}_2 = 300 - 130/3 = 256.7 \text{ mm}$

$\bar{y}_2 = 280 - 130/3 = 236.7 \text{ mm}$

$$A_3 = \frac{\pi d^2}{4} = \frac{\pi}{4}(50)^2 = 1963 \text{ mm}^2$$

$\bar{x}_3 = 80 \text{ mm} \qquad \bar{y}_3 = 80 \text{ mm}$

$A_4 = 1963 \text{ mm}^2 \qquad \bar{x}_4 = 220 \text{ mm} \qquad \bar{y}_4 = 80 \text{ mm}$

$$A = \sum A_i = A_1 - A_2 - A_3 - A_4 = 71{,}620 \text{ mm}^2$$

$$Q_y = \sum \bar{x}_i A_i = \bar{x}_1 A_1 - \bar{x}_2 A_2 - \bar{x}_3 A_3 - \bar{x}_4 A_4$$

$$= 9.842 \times 10^6 \text{ mm}^3$$

$$\bar{x} = \frac{Q_y}{A} = \frac{9.842 \times 10^6}{71{,}620} = 137 \text{ mm} \quad \longleftarrow$$

$$Q_x = \sum \bar{y}_i A_i = \bar{y}_1 A_1 - \bar{y}_2 A_2 - \bar{y}_3 A_3 - \bar{y}_4 A_4$$

$$= 9.446 \times 10^6 \text{ mm}^3$$

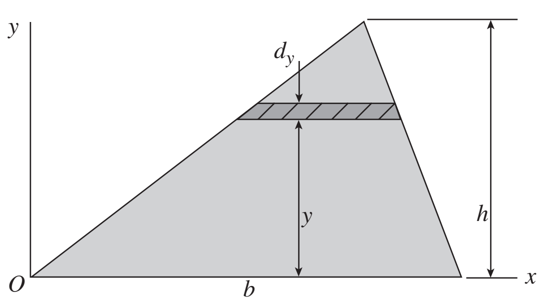
$$\bar{y} = \frac{Q_x}{A} = \frac{9.446 \times 10^6}{71{,}620} = 132 \text{ mm} \quad \longleftarrow$$

## Moments of Inertia

*Problems 12.4-1 through 12.4-4 are to be solved by integration.*

**Problem 12.4-1** Determine the moment of inertia $I_x$ of a triangle of base $b$ and altitude $h$ with respect to its base (see Case 4, Appendix D).

**Solution 12.4-1** **Moment of inertia of a triangle**




Width of element

$$= b\left(\frac{h - y}{h}\right)$$

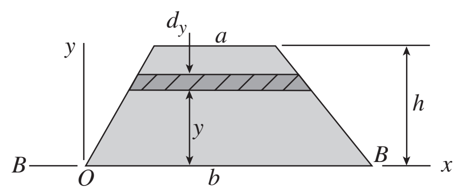
$$dA = \frac{b(h - y)}{h} dy$$

$$I_x = \int y^2 dA = \int_0^h y^2 b \frac{(h - y)}{h} dy$$

$$= \frac{bh^3}{12} \quad \longleftarrow$$

**Problem 12.4-2** Determine the moment of inertia $I_{BB}$ of a trapezoid having bases $a$ and $b$ and altitude $h$ with respect to its base (see Case 8, Appendix D).

**Solution 12.4-2 Moment of inertia of a trapezoid**




Width of element

$$= a + (b - a)\left(\frac{h - y}{h}\right)$$

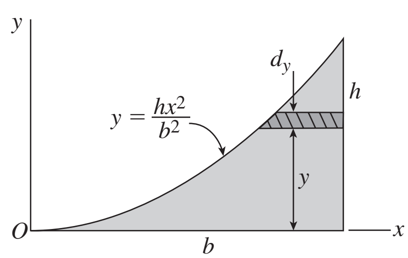
$$dA = \left[a + (b - a)\left(\frac{h - y}{h}\right)\right] dy$$

$$I_{BB} = \int y^2 dA = \int_0^h y^2\left[a + (b - a)\left(\frac{h - y}{h}\right)\right] dy$$

$$= \frac{h^3(3a + b)}{12} \quad \longleftarrow$$

**Problem 12.4-3** Determine the moment of inertia $I_x$ of a parabolic spandrel of base $b$ and height $h$ with respect to its base (see Case 18, Appendix D).

**Solution 12.4-3 Moment of inertia of a parabolic spandrel**




Width of element

$$= b - x = b - b\sqrt{\frac{y}{h}}$$

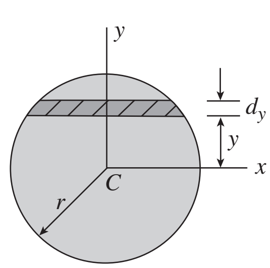
$$= b(1 - \sqrt{y/h})$$

$$dA = b(1 - \sqrt{y/h})\, dy$$

$$I_x = \int y^2 dA = \int_0^h y^2 b\,(1 - \sqrt{y/h})\, dy = \frac{bh^3}{21} \quad \longleftarrow$$

**Problem 12.4-4** Determine the moment of inertia $I_x$ of a circle of radius $r$ with respect to a diameter (see Case 9, Appendix D).

**Solution 12.4-4 Moment of inertia of a circle**




$$\text{Width of element} = 2\sqrt{r^2 - y^2}$$

$$dA = 2\sqrt{r^2 - y^2}\, dy$$

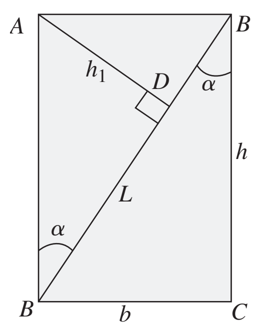
$$I_x = \int y^2 dA = \int_{-r}^{r} y^2(2\sqrt{r^2 - y^2})\, dy$$

$$= \frac{\pi r^4}{4} \quad \longleftarrow$$

*Problems 12.4-5 through 12.4-9 are to be solved by considering the area to be a composite area.*

**Problem 12.4-5** Determine the moment of inertia $I_{BB}$ of a rectangle having sides of lengths $b$ and $h$ with respect to a diagonal of the rectangle (see Case 2, Appendix D).

**Solution 12.4-5 Moment of inertia of a rectangle with respect to a diagonal**




$L$ = length of diagonal $BB$

$L = \sqrt{b^2 + h^2}$

$h_1$ = distance from $A$ to diagonal $BB$ triangle $BBC$:

$$\sin\alpha = \frac{b}{L}$$

Triangle $ADB$: $\sin\alpha = \dfrac{h_1}{h}$ $\quad h_1 = h\sin\alpha = \dfrac{bh}{L}$

$I_1$ = moment of inertia of triangle $ABB$ with respect to its base $BB$

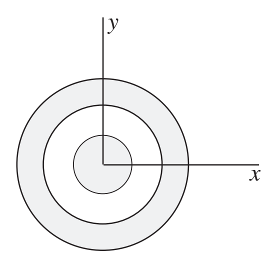
From Case 4, Appendix D:

$$I_1 = \frac{Lh_1^3}{12} = \frac{L}{12}\left(\frac{bh}{L}\right)^3 = \frac{b^3h^3}{12L^2}$$

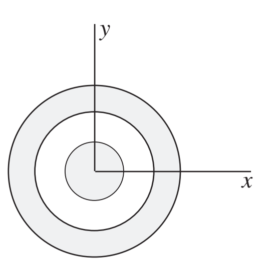
For the rectangle:

$$I_{BB} = 2I_1 = \frac{b^3h^3}{6(b^2 + h^2)} \quad \longleftarrow$$

**Problem 12.4-6** Calculate the moment of inertia $I_x$ for the composite circular area shown in the figure. The origin of the axes is at the center of the concentric circles, and the three diameters are 20, 40, and 60 mm.




**Solution 12.4-6 Moment of inertia of composite area**




Diameters = 20, 40, and 60 mm

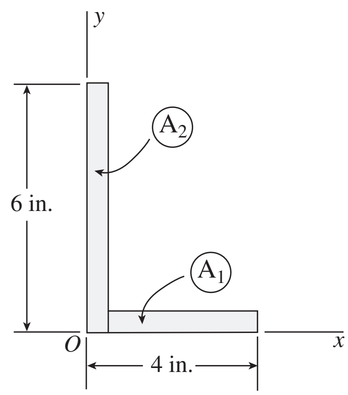
$$I_x = \frac{\pi d^4}{64} \text{ (for a circle)}$$

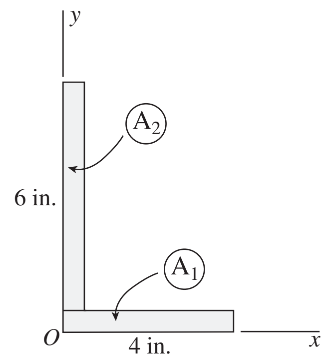
$$I_x = \frac{\pi}{64}[(60)^4 - (40)^4 + (20)^4]$$

$$I_x = 518 \times 10^3 \text{ mm}^4 \quad \longleftarrow$$

**Problem 12.4-7** Calculate the moments of inertia $I_x$ and $I_y$ with respect to the $x$ and $y$ axes for the L-shaped area shown in the figure for Prob. 12.3-7.

**Solution 12.4-7 Moments of inertia of composite area**

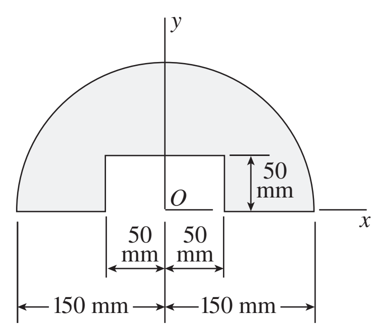


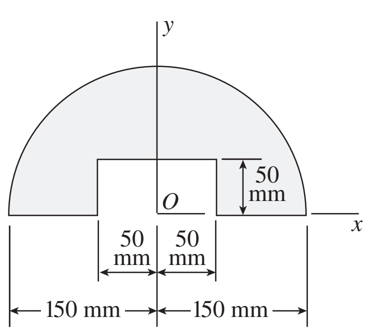

$$I_x = I_1 + I_2$$
$$= \frac{1}{3}(3.5)(0.5)^3 + \frac{1}{3}(0.5)(6)^3$$
$$= 36.1 \text{ in.}^4 \longleftarrow$$
$$I_y = I_3 + I_4$$
$$= \frac{1}{3}(0.5)(4)^3 + \frac{1}{3}(5.5)(0.5)^3$$
$$= 10.9 \text{ in.}^4 \longleftarrow$$

**Problem 12.4-8** A semicircular area of radius 150 mm has a rectangular cutout of dimensions 50 mm × 100 mm (see figure).

Calculate the moments of inertia $I_x$ and $I_y$ with respect to the $x$ and $y$ axes. Also, calculate the corresponding radii of gyration $r_x$ and $r_y$.




**Solution 12.4-8 Moments of inertia of composite area**



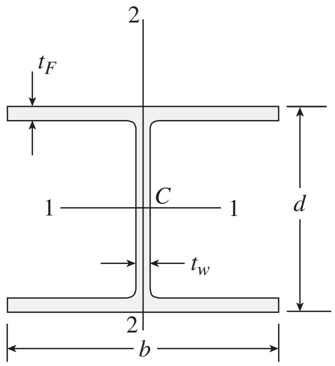

All dimensions in millimeters

$r = 150$ mm $\quad b = 100$ mm $\quad h = 50$ mm

$$I_x = (I_x)_{\text{semicircle}} - (I_x)_{\text{rectangle}} = \frac{\pi r^4}{8} - \frac{bh^3}{3}$$
$$= 194.6 \times 10^6 \text{ mm}^4 \longleftarrow$$
$$I_y = I_x \longleftarrow$$
$$A = \frac{\pi r^2}{2} - bh = 30.34 \times 10^3 \text{ mm}^2$$
$$r_x = \sqrt{I_x/A} = 80.1 \text{ mm} \longleftarrow$$
$$r_y = r_x \longleftarrow$$

**Problem 12.4-9** Calculate the moments of inertia $I_1$ and $I_2$ of a W 16 × 100 wide-flange section using the cross-sectional dimensions given in Table E-l, Appendix E. (Disregard the cross-sectional areas of the fillets.) Also, calculate the corresponding radii of gyration $r_1$ and $r_2$, respectively.

**Solution 12.4-9** **Moments of inertia of a wide-flange section**




W 16 × 100 $d = 16.97$ in.

$t_w = t_{web} = 0.585$ in.

$b = 10.425$ in.

$t_F = t_{Flange} = 0.985$ in.

All dimensions in inches.

$$I_1 = \frac{1}{12}bd^3 - \frac{1}{12}(b - t_w)(d - 2t_F)^3$$

$$= \frac{1}{12}(10.425)(16.97)^3 - \frac{1}{12}(9.840)(15.00)^3$$

$$= 1478 \text{ in.}^4 \qquad \text{say,} \quad I_1 = 1480 \text{ in.}^4 \longleftarrow$$

$$I_2 = 2\left(\frac{1}{12}\right)t_F b^3 + \frac{1}{12}(d - 2t_F)t_w^3$$

$$= \frac{1}{6}(0.985)(10.425)^3 + \frac{1}{12}(15.00)(0.585)^3$$

$$= 186.3 \text{ in.}^4 \qquad \text{say,} \quad I_2 = 186 \text{ in.}^4 \longleftarrow$$

$$A = 2(bt_F) + (d - 2t_F)t_w$$

$$= 2(10.425)(0.985) + (15.00)(0.585)$$

$$= 29.31 \text{ in.}^2$$

$$r_1 = \sqrt{I_1/A} = 7.10 \text{ in.} \longleftarrow$$

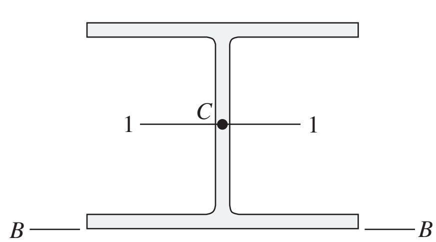
$$r_2 = \sqrt{I_2/A} = 2.52 \text{ in.} \longleftarrow$$

Note that these results are in close agreement with the tabulated values.

## Parallel-Axis Theorem

**Problem 12.5-1** Calculate the moment of inertia $I_b$ of a W 12 × 50 wide-flange section with respect to its base. (Use data from Table E-l, Appendix E.)

**Solution 12.5-1** **Moment of inertia**



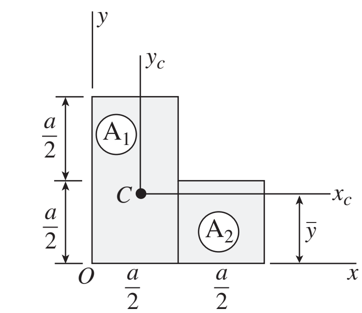

W 12 × 50 $I_1 = 394 \text{ in.}^4$ $A = 14.7 \text{ in.}^2$
$d = 12.19$ in.

$$I_b = I_1 + A\left(\frac{d}{2}\right)^2$$

$$= 394 + 14.7(6.095)^2 = 940 \text{ in.}^4 \longleftarrow$$

**Problem 12.5-2** Determine the moment of inertia $I_c$ with respect to an axis through the centroid $C$ and parallel to the $x$ axis for the geometric figure described in Prob. 12.3-2.

**Solution 12.5-2 Moment of inertia**




From Prob. 12.3-2:

$A = 3a^2/4$

$\bar{y} = 5a/12$

$$I_x = \frac{1}{3}\left(\frac{a}{2}\right)(a^3) + \frac{1}{3}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)^3 = \frac{3a^4}{16}$$

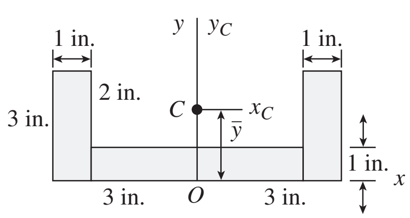
$$I_x = I_{x_C} + A\bar{y}^2$$

$$I_c = I_{x_C} = I_x - A\bar{y}^2 = \frac{3a^4}{16} - \frac{3a^2}{4}\left(\frac{5a}{12}\right)^2$$

$$= \frac{11a^4}{192} \quad \leftarrow$$

**Problem 12.5-3** For the channel section described in Prob. 12.3-3, calculate the moment of inertia $I_{x_c}$ with respect to an axis through the centroid $C$ and parallel to the $x$ axis.

**Solution 12.5-3 Moment of inertia**




From Prob. 12.3-3:

$A = 10.0$ in.$^2$

$\bar{y} = 1.10$ in.

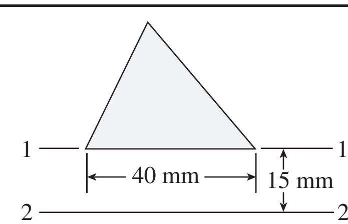
$I_x = 1/3(4)(1)^3 + 2(1/3)(1)(3)^3 = 19.33$ in.$^4$

$I_x = I_{x_C} + A\bar{y}^2$

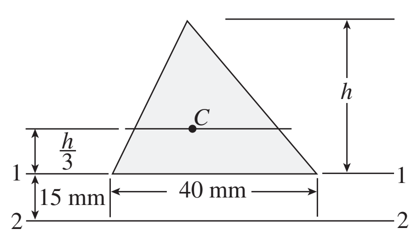
$$I_{x_C} = I_x - A\bar{y}^2 = 19.33 - (10.0)(1.10)^2$$

$= 7.23$ in.$^4$ $\leftarrow$

**Problem 12.5-4** The moment of inertia with respect to axis 1-1 of the scalene triangle shown in the figure is $90 \times 10^3$ mm$^4$. Calculate its moment of inertia $I_2$ with respect to axis 2-2.




**Solution 12.5-4 Moment of inertia**




$b = 40$ mm $\quad I_1 = 90 \times 10^3$ mm$^4$ $\quad I_1 = bh^3/12$

$$h = \sqrt[3]{\frac{12I_1}{b}} = 30 \text{ mm}$$

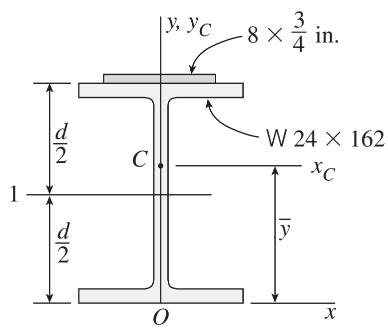
$I_c = bh^3/36 = 30 \times 10^3$ mm$^4$

$$I_2 = I_c + Ad^2 = I_c + (bh/2)\,d^2 = 30 \times 10^3$$

$$+ \frac{1}{2}(40)\,(30)\,(25)^2 = 405 \times 10^3 \text{ mm}^4 \quad \leftarrow$$

**Problem 12.5-5** For the beam cross section described in Prob. 12.3-5, calculate the centroidal moments of inertia $I_{x_c}$ and $I_{y_c}$ with respect to axes through the centroid $C$ such that the $x_c$ axis is parallel to the $x$ axis and the $y_c$ axis coincides with the $y$ axis.

**Solution 12.5-5 Moment of inertia**




From Prob. 12.3-5:

$\bar{y} = 13.94$ in.

W 24 × 162 $\quad d = 25.00$ in. $\quad d/2 = 12.5$ in.

$I_1 = 5170$ in.$^4$ $\quad A = 47.7$ in.$^2$

$I_2 = I_y = 443$ in.$^4$

$$I'_{x_c} = I_1 + A(\bar{y} - d/2)^2 = 5170 + (47.7)(1.44)^2 = 5269 \text{ in.}^4$$

$$I'_{y_c} = I_2 = 443 \text{ in.}^4$$

PLATE

$$I''_{x_c} = 1/12(8)(3/4)^3 + (8)(3/4)(d + 3/8 - \bar{y})^2$$
$$= 0.2813 + 6(25.00 + 0.375 - 13.94)^2$$
$$= 0.2813 + 6(11.44)^2 = 785 \text{ in.}^4$$
$$I''_{y_c} = 1/12(3/4)(8)^3 = 32.0 \text{ in.}^4$$

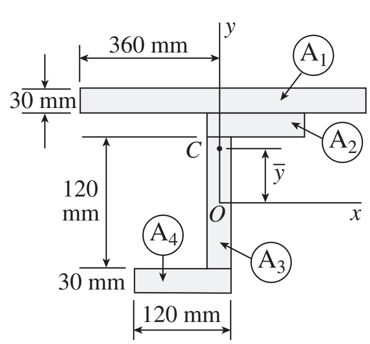
ENTIRE CROSS SECTION

$$I_{x_c} = I'_{x_c} + I''_{x_c} = 5269 + 785 = 6050 \text{ in.}^4 \longleftarrow$$

$$I_{y_c} = I'_{y_c} + I''_{y_c} = 443 + 32 = 475 \text{ in.}^4 \longleftarrow$$

**Problem 12.5-6** Calculate the moment of inertia $I_{x_c}$ with respect to an axis through the centroid $C$ and parallel to the $x$ axis for the composite area shown in the figure for Prob. 12.3-6.

**Solution 12.5-6 Moment of inertia**




From Prob. 12.3-6:

$\bar{y} = 52.50$ mm $\quad t = 30$ mm $\quad A = 21{,}600$ mm$^2$

$A_1$: $I_x = 1/12(360)(30)^3 + (360)(30)(105)^2 = 119.9 \times 10^6$ mm$^4$

$A_2$: $I_x = 1/12(120)(30)^3 + (120)(30)(75)^2 = 20.52 \times 10^6$ mm$^4$

$A_3$: $I_x = 1/12(30)(120)^3 = 4.32 \times 10^6$ mm$^4$

$A_4$: $I_x = 20.52 \times 10^6$ mm$^4$

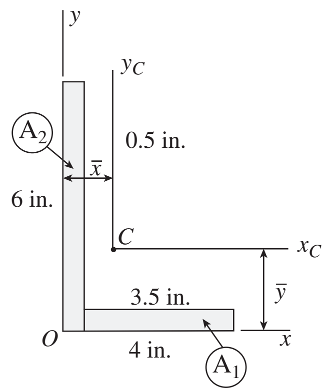
ENTIRE AREA:

$$I_x = \sum I_x = 165.26 \times 10^6 \text{ mm}^4$$

$$I_{x_c} = I_x - A\bar{y}^2 = 165.26 \times 10^6 - (21{,}600)(52.50)^2 = 106 \times 10^6 \text{ mm}^4 \longleftarrow$$

**Problem 12.5-7** Calculate the centroidal moments of inertia $I_{x_c}$ and $I_{y_c}$ with respect to axes through the centroid $C$ and parallel to the $x$ and $y$ axes, respectively, for the L-shaped area shown in the figure for Prob. 12.3-7.

**Solution 12.5-7 Moments of inertia**




From Prob. 12.3-7:

$t = 0.5$ in. $\quad A = 4.75$ in.$^2$
$\bar{y} = 1.987$ in.

$\bar{x} = 0.9869$ in.

From Problem 12.4-7:

$I_x = 36.15$ in.$^4$

$I_y = 10.90$ in.$^4$

$I_{x_c} = I_x - A\bar{y}^2 = 36.15 - (4.75)\,(1.987)^2$

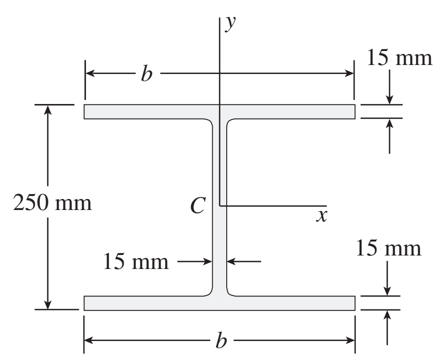
$= 17.40$ in.$^4$ ←

$I_{y_c} = I_y - A\bar{x}^2 = 10.90 - (4.75)(0.9869)^2$

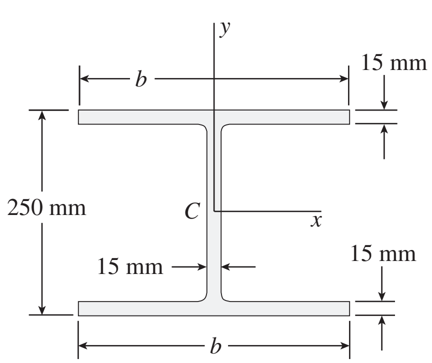
$= 6.27$ in.$^4$ ←

**Problem 12.5-8** The wide-flange beam section shown in the figure has a total height of 250 mm and a constant thickness of 15 mm.

Determine the flange width $b$ if it is required that the centroidal moments of inertia $I_x$ and $I_y$ be in the ratio 3 to 1, respectively.




**Solution 12.5-8 Wide-flange beam**




$t = 15$ mm $\qquad b =$ flange width

All dimensions in millimeters.

$$I_x = \frac{1}{12}(b)(250)^3 - \frac{1}{12}(b-15)(220)^3$$
$$= 0.4147 \times 10^6 b + 13.31 \times 10^6 \text{ (mm)}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(15)(b)^3 + \frac{1}{12}(220)(15)^3$$
$$= 25b^3 + 61{,}880 \text{ (mm}^4)$$

Equate $I_x$ to $3I_y$ and rearrange:

$7.5b^3 - 0.4147 \times 10^6 b - 13.12 \times 10^5 = 0$

Solve numerically:

$b = 250$ mm ←